Retirement Ruin and the Sequencing of Returns

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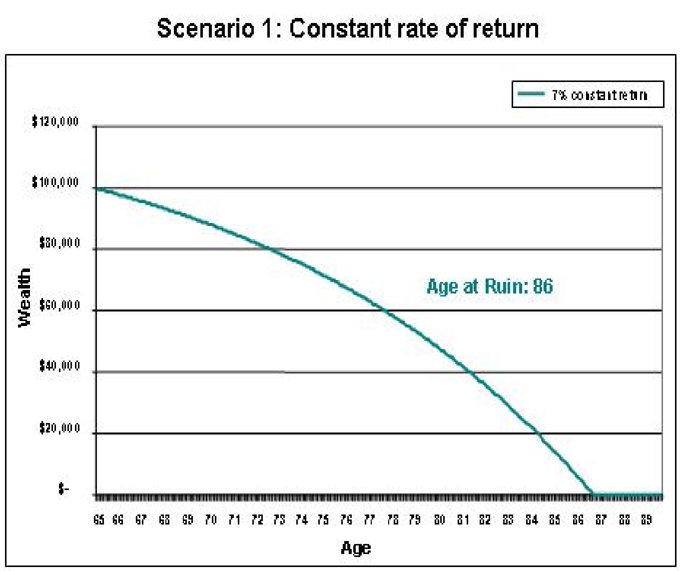
The cold harsh calculus of retirement income tells us with unwavering accuracy exactly how long a nest egg will last under fixed withdrawals and known returns. In a so-called deterministic world one doesn't require spinning roulette wheels or computer simulations to back-out your date with ruin.

For example if your current $100,000 portfolio is subjected to monthly withdrawals of $750 -- which is $9,000 annually -- and is earning a nominal rate of 7% per year (a.k.a. 0.5833% per month), your nest egg will be exhausted within month number 259. Start this doomed process at age 65 and you will become ruined half-way through age 86. Figure #1 illustrates the smooth and predictable path your portfolio will take on its way to zero.

Figure #1:

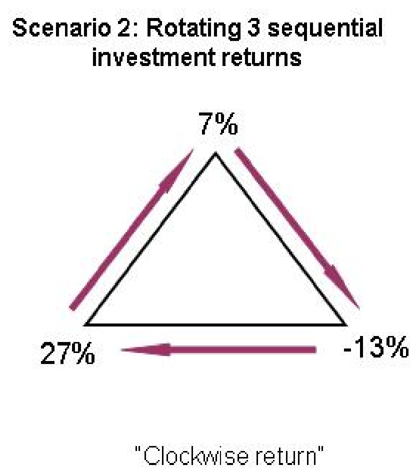


We know this inevitable date with destiny with absolute certainty since the textbook equation $(1-(1+k)^{-n})/k$ teaches us that the present value of $750 for $n = 258.59$ periods under a periodic rate of $k = 0.005833$ is exactly $100,000. Ergo, the $100,000 will only last until age 86.5.

Of course, if you plan to withdraw a lower $625 per month -- which is $7,500 per year -- the money runs out by month 466 and the nest egg lasts beyond the mythical age of 100 for the same 65 year-old retiree. The present value of $625 paid over 465.59 periods under a periodic rate of 0.5833% is also $100,000.

The question we would like to investigate in this brief article is as follows. What happens if the hypothetical 65 year-old retiree does not earn a constant 7% each and every year but instead earns an arithmetic average 7% over his or her retirement? How variable is the final outcome and what does it depend on?

Figure #2:

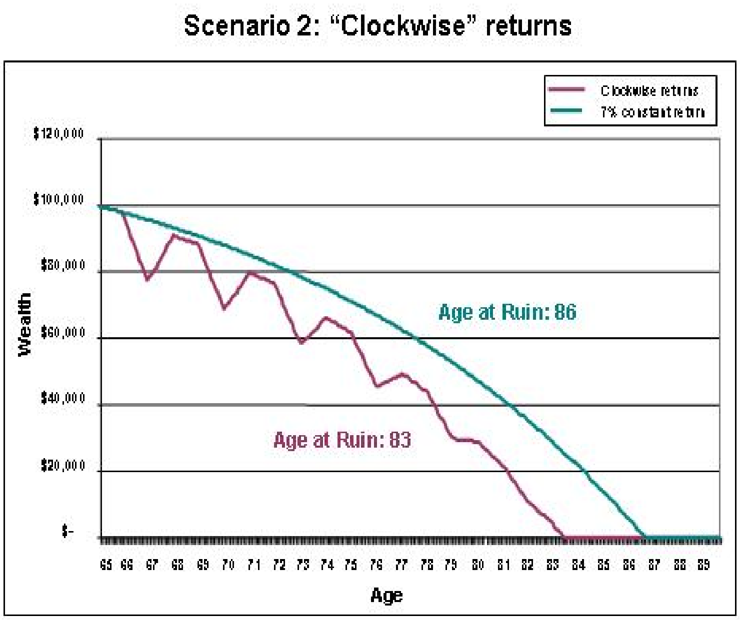


To put some structure on the problem – since there are so many ways to generate an average return of 7% -- imagine that the annual investment returns are generated in a cyclical and systematic manner. Figure #2 illustrates how it works. During the first year of retirement the portfolio earns 7%. In the second year of retirement it earns -13% and in the third year of retirement it earns 27%. By construction, the arithmetic average of these numbers is exactly 7% and each month we plan on withdrawing the same $750 as in the earlier case. Then, in the fourth year we start the cycle again. First the portfolio earns 7%, then -13% and then 27%.

This cyclical process continues in three-year increments until the nest-egg is exhausted and the money runs out. Do you think you will get ruined earlier or later than the prior case where returns were a smooth 7% each and every year? If you think the answer is earlier, you are right. Indeed, since you started retirement on the "wrong foot" the date with zero occurs a full 3 years earlier, or at age 83. The 27% return in your third, sixth, ninth, etc, year of retirement isn't enough to offset the minus 13% returns in the second, fifth, eight, etc, year of retirement. (This is akin to this year's 20% bull market failing to undo the damage of last year's 20% bear market.)

Note that our answer is obtained and can be computed with just as much accuracy as the previous case, although you can't use a simple formula for the present value. Instead, you must do this manually or by hand. A simple spreadsheet in Excel will do the trick (and is attached to this article) and Figure #3 illustrates the result graphically.

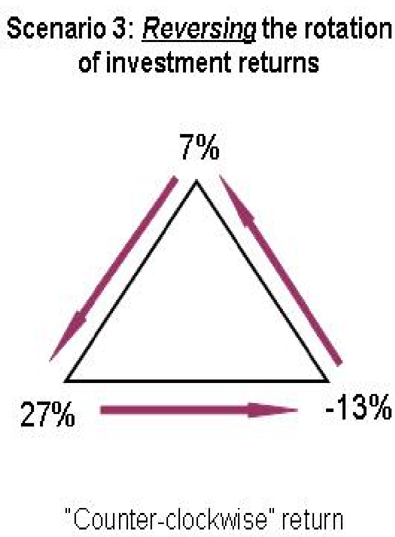
Figure #3



Start with $100,000 and force it to earn 0.5833% in the first month. Then, withdraw $750 and have the remaining sum earn the same 0.5833% for the next month. Do this for 12 months and then repeat for 12 months under an investment return of -1.0833% per month, which is nominal -13% per year. Finally, repeat for 12 months under an investment return of 2.2500% per month, which is a nominal 27% per year. Every 36 months the pattern repeats itself. Start with twelve 0.5833% numbers, then twelve -1.0833% numbers and finally twelve 2.2500% numbers. You should have a very long column of returns which mimics the picture in Figure #3, with the account ultimately reaching zero shortly after your 83rd birthday. In this case, an average of 7% is <u>worse</u> than a 7% every year.

Now, what happens if you reverse the triangle and instead start in the other direction? In other words, what happens if you earn 7%, then 27% and then -13% over and over again? Figure #4 displays the same triangle, but with the arrows going in the other direction.

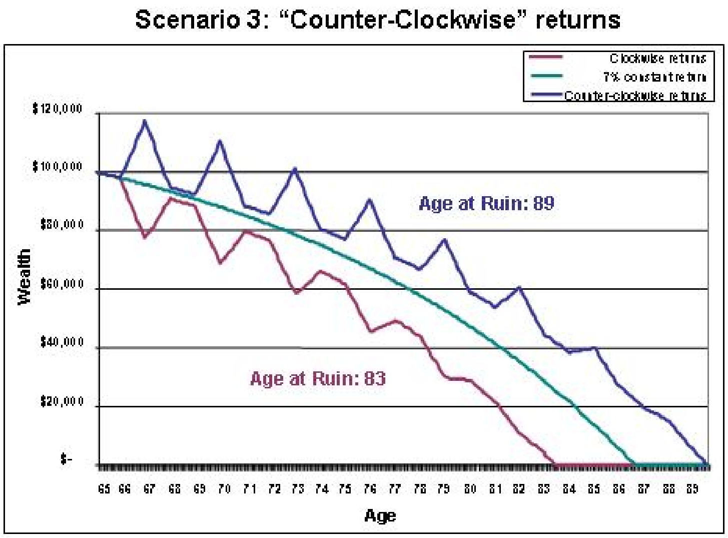
Figure #4



The arithmetic average investment return is the same 7% regardless of what side of the triangle you start retirement earnings and withdrawals. However, this time around, the money runs out at age 89.5 as opposed to age 83.33 or 86.5

In this case, an average of 7% is better than 7% every year.

Figure #5



The variance in outcomes would have been even greater if I started with -13% or 27% as opposed to the same 7%. For example, if the sequence was -13%, 7% and then 27%, the age of ruin would be 81. This peculiar phenomenon is unique to the distribution phase of the lifecycle. In the accumulation phase – as money is being added to the account on an ongoing basis – it is impossible to exhaust the account no matter how poor the returns. Also, remember that: $(1.07)(1.27)(0.87) = (1.07)(0.87)(1.27)$.

Finally, Table #1 summarizes the impact of the various sequences on the ruin age as well as the variation in months between the given sequence and the baseline case of 7% each and every year of retirement. Note that this sequencing gap can get quite large. There is a 14 year gap between repeating the sequence {-13%,7%,27%} versus {27%,7%,-13%}.

Table #1

What stop did you get on the retirement merry-go-round?

Return Sequence	*Ruin Age*	*+/- Months*
+7%, +7%, +7%...	86.50	
+7%, -13%, +27%...	83.33	-38
+7%, +27%, -13%...	89.50	+36
-13%, +7%, +27%...	81.08	-65
+27%, +7%, -13%...	94.92	+101

***Assumes $9,000 spending per year**.

What do we learn? First of all, arithmetic averages can be a deceiving measure of central tendency when it comes to investment returns while withdrawing. The arithmetic average of -13%, 7% and 27% is exactly 7%. However, the geometric average of these three numbers is $((1-0.13)(1+0.07)(1+0.27))^{1/3}-1=5.6\%$, which provides a more pessimistic (but more accurate) indication of the risks that lie ahead. Remember that the greater the gap between your portfolio's arithmetic and geometric mean the greater the chances of an early ruin, all else being equal.

More importantly, this is yet another indication of how fragile the first few investment years of withdrawals can really be…and why they should be protected*. Don't leave your retirement income at the mercy of a spinning merry-go-round!*